OPTIMAL SENSOR SELECTION IN A MULTISTATIC PASSIVE RADAR SCENARIO

Pietro Stinco, Maria Sabrina Greco, Fulvio Gini, Lucio Verrazzani

Department of “Ingegneria dell’Informazione”, University of Pisa, Pisa, Italy

ABSTRACT

Multistatic passive and active systems can offer many advantages in terms of coverage and accuracy in the estimation of target signal parameters but unfortunately their performance are heavily sensitive to the position of receivers (RX) and transmitters (TX) with respect to the target trajectory. In this work, we describe an algorithm for selecting, along the trajectory of the tracked target, the pair TX-RX with the best estimation accuracy of target range and velocity.

1. INTRODUCTION

Recently, great interest has been devoted to systems making use of illuminators of opportunity, such as broadcast or communications signals, tracking targets by range and Doppler information. These techniques are known as Passive Coherent Location (PCL), and have the advantage that the receivers do not need any transmitter hardware of their own, are completely passive, and hence undetectable. In PCL systems multiple receiver sites are used to provide several different bistatic channels of observation, leading to an increase in the information on a particular area of surveillance. The information gain obtained through this spatial diversity can give rise to a number of advantages in typical radar functions. However, the performance of each bistatic channel heavily depends upon the geometry of the scenario and the position of the target with respect to each receiver and transmitter.

As known, geometry factors play an important role in the shape of the bistatic ambiguity function which is often used to measure the possible global resolution and large error properties of the target parameters estimates. In this work, we exploit the relation between the Ambiguity Function (AF) and the Cramér-Rao Lower Bound (CRLB) to calculate the bistatic CRLBs of target range and velocity of each TX-RX pair as a function of the target kinematic parameters. Using the information gained through this calculation, we approach the problem of selecting the optimum sensor of the system, that is the TX-RX pair which exhibits the best estimation accuracy of the target velocity and/or range.

The analyzed PCL system is composed by a single non co-operative frequency modulated (FM) commercial radio station as transmitter of opportunity and four receivers.

2. BISTATIC AMBIGUITY FUNCTION

In our study we modeled the complex envelope of the signal transmitted by the non co-operative FM commercial radio station with the unitary power pulse given by:

\[ u(t) = \frac{1}{\sqrt{T}} e^{j\beta \sin(2\pi \nu t)} \text{rect}(\frac{t}{T}) \]  

(1)

That is a pulse which instantaneous frequency is a sinusoidal oscillation. In particular, \( T \) is the observation time, \( \beta \) is the modulation index and \( 1/f_0 \) is the period of the instantaneous frequency. In other words, we assumed that, during the observation time, the modulating signal transmitted by a radio station can be approximated by a sinusoidal oscillation. This can be justified considering that in a typical FM radio, the program content is speech and/or music, which are often modelled as periodic vibrations. Moreover, the chosen signal is a mathematically tractable model that makes it feasible to study the analyzed scenario rigorously. As widely known, the complex ambiguity function (CAF) of a pulse \( u(t) \) represents the response of a filter matched to a given finite energy signal when the signal is received with a delay \( \tau \) and a Doppler shift \( \nu \) relative to the nominal values \( \tau_0 \) and \( \nu_0 \) expected by the receiver. Therefore, the CAF definition is [1]:

\[ X(\tau_\nu, \tau_\nu, \nu_\nu, \nu_\nu) = \int_{-\infty}^{\infty} u(t-\tau_\nu) u^*(t-\nu_\nu) e^{-j2\pi\nu_\nu \nu_\nu} dt \]  

(2)

In the monostatic case there is a linear relation between \( \tau_\nu \) and \( \nu_\nu \) and the target range position \( R_\nu \) and radial velocity \( V_\nu \), i.e. \( \tau_\nu = 2R_\nu/c \) and \( \nu_\nu = 2V_\nu/c \), where \( f_C \) is the carrier frequency of the reference signal \( u(t) \). Similar relations hold for \( \tau_\nu \) and \( \nu_\nu \). The Ambiguity Function (AF) is the absolute value of the CAF. Clearly, \( |X(\tau_\nu, \tau_\nu, \nu_\nu, \nu_\nu)| \) is maximum for \( \tau_\nu = \tau_\nu \) and \( \nu_\nu = \nu_\nu \).

Based upon definition (2) we can calculate the monostatic CAF for the signal in (1). In particular, after some manipulation, it is possible to write

\[ X(\tau, \nu) = \sum_{n=-\infty}^{\infty} e^{j(\nu-n)\nu_\nu} J_n(\beta) J_n(\beta) e^{j\nu_\nu \nu_\nu} |X| \left( \frac{\nu}{\nu T} \right) \text{rect}(\frac{\tau}{2T}) \]  

(3)

with

\[ |X| = \sin^2 \left( \frac{\pi \nu (T-|\nu|)}{\nu T} \right) \text{rect}(\frac{\tau}{2T}) \]  

(4)
where we set $\tau=\tau_u-\tau_v$, $v=v_u-v_v$ and $J_n(\beta)$ is the $n^{th}$ order Bessel function of the first kind.

Unlike the AF, which provides information on the global resolution, the CRLBs are a local measure of estimation accuracy. Anyway, both can be used to assess the error properties of the estimates of the signal parameters. In [3] the author derived a relationship between CRLB and AF, which has been successfully used in the analysis of passive and active arrays [1],[4],[5]. In the monostatic configuration, [3] claims that for the Fisher Information Matrix (FIM) the following relationship holds:

$$
J_u(\tau_v,v_v) = -2SNR \begin{bmatrix}
\frac{\partial^2}{\partial \tau^2} X(\tau,v) \\
\frac{\partial^2}{\partial \tau \partial v} X(\tau,v) \\
\frac{\partial^2}{\partial v^2} X(\tau,v)
\end{bmatrix}_{\tau=-\bar{\tau},v=0} \tag{5}
$$

It is apparent that the FIM depends on both the signal-to-noise power ratio at the receiver SNR and the second derivatives of the AF, that is the sharpness of the ambiguity function mainlobe.

After some algebra, it is possible to verify that [5]:

$$
[J_u]_{1,1} = 2SNR \frac{B^2 \pi f_c}{T} [2\pi f_c T + \sin(2\pi f_c T) \cos(2\varphi)]
$$

$$
[J_u]_{1,2} = 2SNR \frac{\pi^3 T^2}{3}
$$

$$
[J_u]_{2,2} = -2SNR \frac{2\beta \sin(\varphi)}{T f_c} [\pi f_c T \cos(\pi f_c T) - \sin(\pi f_c T)]
$$

From eq. (5) the CRLBs follow:

$$
\text{CRLB}(\tau_v) = [J_u(\tau_v,v_v)]_{1,1} \quad \text{and} \quad \text{CRLB}(v_v) = [J_u(\tau_v,v_v)]_{2,2}
$$

Assuming $T=k/f_c$ ( $k \in \mathbb{N}$ ) and considering that $\pi^2k^3/3>\sin^2(\varphi)$, it is easy to verify that the Root of CRLB($v_v$) is inversely proportional to the observation time $T$, while the Root of CRLB($\tau_v$) is inversely proportional to $B_c = 2f_c$, that is the Carson’s Bandwidth of the emitted signal. From this result it is interesting to observe that the best performance is obtained with modulating signals with high spectral content, such as rock music, and poorest performance is obtained with slow varying modulating signals, such as speech modulation. To obtain the expression of the bistatic ambiguity function, we must replace $\tau_u$ and $v_u$ in (3)-(4) with the following [2],[4],[5]:

$$
\tau_u = R_x + \sqrt{R_x^2 + L^2 + 2R_x L \sin \theta_s} \quad \tag{9}
$$

and

$$
v_v = \frac{2L \nu}{c} \sqrt{\frac{1}{2} + \frac{R_x + L \sin \theta_s}{\sqrt{2R_x^2 + L^2 + 2R_x L \sin \theta_s}}} \quad \tag{10}
$$

where $R_x$ is the range from receiver to target, $L$ is the baseline between the transmitter and the receiver, $\theta_s$ is the look angle of the receiver and $v_v$ is the component of the target velocity in the direction of the bistatic bisector, i.e. the bisector of the angle at the vertex of the bistatic triangle which represents the target. In the above equations we used the cosine low relation $R_t^2 = R_x^2 + L^2 + 2R_x L \sin \theta_s$, which gives the range from transmitter to target $R_t$, as a function of the range from receiver to target and the look angle of the receiver. In the bistatic configuration we should express the ambiguity function in terms of the bistatic $\tau(R_x,\theta_s,L)$ and $v(R_x,V_x,\theta_s,L)$, and derive it with respect to the useful parameters $R_x$ and $V_x$, as follows:

$$
J_x(R_x,V_x) = -2SNR \begin{bmatrix}
\frac{\partial^2}{\partial R_x^2} X(R_x,V_x) \\
\frac{\partial^2}{\partial R_x \partial V_x} X(R_x,V_x) \\
\frac{\partial^2}{\partial V_x^2} X(R_x,V_x)
\end{bmatrix}_{\tau=R_x,\nu=V_x}
$$

Using the derivative chain rule and letting $R=R_x$ and $V=V_x$ after some algebra it is possible to verify that [4],[5]:

$$
[J_x]_{1,1} = [J_u]_{1,1} \left( \frac{\partial^2}{\partial R_x^2} \right)^2 + 2[J_u]_{1,2} \frac{\partial^2}{\partial R_x \partial R_x} + [J_u]_{2,2} \left( \frac{\partial^2}{\partial R_x^2} \right)^2 \quad \tag{11}
$$

$$
[J_x]_{1,2} = [J_u]_{1,1} \frac{\partial^2}{\partial V_x \partial V_x} + [J_u]_{1,2} \frac{\partial^2}{\partial R_x \partial V_x} + [J_u]_{2,2} \frac{\partial^2}{\partial V_x^2} \quad \tag{12}
$$

$$
[J_x]_{2,2} = [J_u]_{1,1} \frac{\partial^2}{\partial R_x \partial R_x} + [J_u]_{1,2} \frac{\partial^2}{\partial R_x \partial V_x} + [J_u]_{2,2} \frac{\partial^2}{\partial V_x \partial V_x} \quad \tag{13}
$$

From the last equation it is clearly apparent that the local accuracy in the bistatic case depends not only on the transmitted waveform but also on the bistatic geometry. For $L=0$ the bistatic CRLBs coincide with the monostatic CRLBs.

### 3. TX-RX PAIR SELECTION

In our scenario we considered a surveillance map of dimension $L_x=100km$ and $L_y=100km$ where we placed 1 transmitter and 4 receivers. We placed the transmitter at coordinates $T=(L_x/2, L_y/2)$ and the receivers at coordinates $R^{(1)}=(L_x/4, L_y/2)$, $R^{(2)}=(L_x/2, 3L_y/4)$, $R^{(3)}=(L_x/2, L_y/4)$ and $R^{(4)}=(3L_x/4, L_y/2)$. So, there are 4 pairs of TX–RX that we considered as independent bistatic systems. The carrier frequency of the system was fixed to $f_c = 100MHz$, which falls within the VHF part of the radio spectrum in which FM is used for broadcasting. Considering that FM commercial radio stations use bandwidth of about 150kHz and the emitted signal belongs to the range of audible frequency, according to Carson’s rule, we fixed $f_0=15kHz$ and $\beta=5$. 
Figure 1 – (a) Optimum pair map for target range estimation; (b) Optimum pair map for target velocity estimation; (c) RCRLB of the target range [dBm] provided selecting the best TX-RX pair; (d) RCRLB of the target velocity [dBm/sec] provided selecting the best TX-RX pair.

We also set $T=20/f_0$ and $\varphi=\pi/2$. In the analyzed scenario we considered an omni-directional antenna for the transmitter and directive antennas for the receivers. In particular, we considered a receiver antenna power gain constant for all the directions with a notch towards the transmitter. We choose this receiver antenna power gain because in a real case scenario the receiver antennas are designed in order to maximize the power ratio between the target echo and the Line Of Sight co-channel interference.

For each point of the analyzed area and for each of the 4 bistatic channels we calculated the Root of the CRLBs (RCRLBs) on the target range and velocity estimation accuracy. In particular, we assumed that, in each point of the analyzed area, the target has a velocity vector aligned to the $x$ axis with intensity 250 m/sec.

The RCRLBs of target range and velocity are a function of the range from the receiver to the target $R_R$, the baseline $L$, the look angle of the receiver $\theta_R$, the radial velocity $V_a$, and the Signal-to-Noise Power Ratio (SNR). Bistatic geometry also affects the received echo power, because the path loss factor is $(R_R R_T)^2$. In particular, the SNR can be written as $\text{SNR}=G(\theta_R)\text{SNR}_C(L_x^2+L_y^2)^2/(R_R R_T)^2$, where $G(\theta_R)$ is the receiver antenna power gain and $\text{SNR}_C$ is a constant parameter. We assumed that $\text{SNR}_C=20\text{dB}$. Therefore, after calculating the RCRLBs of each TX-RX pair, it is possible to evaluate, for each point of the analyzed area, the TX-RX pair having the best performance.

Figures 1-(a) and 1-(b) show, in a colour coded map, the transmitter-receiver pair which has the minimum RCRLB for each point of the analyzed area. The color-map of these figures is divided into 4 colours, each of which is associated with one of the 4 analyzed bistatic systems.

These maps are very similar, but it is also possible to built a cost function using a weighted combination of both CRLBs for selecting the best TX-RX pair. These results can be used for the dynamical selection of the TX-RX signals for the tracking of a radar target moving along a trajectory in a multistatic scenario.

4. REFERENCES


