REFRACTED PROPAGATION DISTORTION AND ITS CORRECTION FOR AIRBORNE RADAR

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ABSTRACT

This paper is devoted to the study of mathematical models for the refracted propagation effects and their correction for airborne radar tracking. Given an atmosphere of varying refracted index, the path of the EM radiation is curved and this causes bias errors on range and elevation radar measurements. First, we model these atmospheric errors and then we generate the corrupted measures. Second, we process the synthetic data to evaluate the induced bias of range and elevation errors. Finally, an algorithm, based on the Kalman Filter (KF) is proposed in order to reduce such errors and improve tracking performance. Both theoretical and numerical results are described.

1. INTRODUCTION

In the last few years technologies for identification and authentication, border security and controlled access to critical infrastructures have became a very important concern. As pointed out in [1], airborne radar is a fundamental sensor in an integrated surveillance system. In this paper, we focus our attention on the atmospheric propagation effects on the airborne radar EM signals. In Section 2, a scenario based on [1] is chosen. In Section 3, a mathematical model to describe and calculate the EM radar signal curvature is discussed. Using such model, the range and elevation bias errors are evaluated and the radar corrupted measures are then generated according to this model. In Section 4, two algorithms for the estimate of the atmospheric errors from the corrupted radar measures are discussed and numerically investigated. Finally, in Section 5, we propose an algorithm, based on the Kalman Filter (KF), to reduce the atmospheric bias errors during the tracking procedures. Simulation results concerning the performance of the proposed algorithm are presented in Section 6. Our conclusions are collected in Section 7.

2. DESCRIPTION OF THE SCENARIO

In our study we have used a 3D Airborne Early Warning Radar (AEWR). Such radar has a range accuracy $\sigma_r$ of 2.4 m and an azimuth and elevation accuracy, $\sigma_{\theta}$ and $\sigma_{\varepsilon}$ respectively, of 0.25°. The scan time is of 1 second. Keeping in mind that airborne radar must be able to cover a wide area, in our simulation we chose a quasi-ellipsoidal race for the air platform centered at the centre of the area to be monitored. The flight height is of 915 m.

Figure 1. Geometry of the scenario.

3. THE ATMOSPHERIC PROPAGATION MODEL

The atmospheric refraction is mostly caused by the first atmospheric layer called troposphere, extending from the Earth’s surface to about 8 km. The variation of the refractive index, as function of the altitude, gives rise to a geometrical distortion of the EM signal, modifying range and elevation measurements (Fig. 2). In order to derive a mathematical model for the EM propagation, we make two assumptions: (A.1) the atmospheric temperature and humidity, then the refractive index, vary only with the altitude; and (A.2) a spherical model with radius $r_0$ for the Earth’s geoid is assumed. As shown in Fig. 2, the corrupted radar measurement vector $\mathbf{z}[k]$ can be defined in spherical coordinates:

$$\mathbf{z}'[k] = \mathbf{z}[k] + \mathbf{\mu}[k], \quad (1)$$

where $\mathbf{\mu}[k] = (\Delta \rho[k], 0, \Delta \varepsilon[k])^T$ is a vector of the range and elevation bias errors. The vector $\mathbf{z}[k]$ represents the noisy radar measures, defined as:

$$\mathbf{z}[k] = (\rho[k], \theta[k], \varepsilon[k])^T + \mathbf{\nu}[k] \quad (2)$$

where $\mathbf{\nu}[k]$ is the measurement noise vector, that is a zero-mean, Gaussian distributed random vector with diagonal covariance matrix given by $\mathbf{C}_\nu = \text{diag}(\sigma_\rho^2, \sigma_{\theta}^2, \sigma_{\varepsilon}^2)$. In the following, we define as corrupted the position vector af-
fected by the only geometrical distortion (not considering the measurement noise).

4. EVALUATION OF ATMOSPHERIC ERRORS FROM CORRUPTED MEASURES

4.1. Evaluation of the elevation error

In order to estimate the elevation error from the corrupted radar measures, we use the procedure shown in [2] and [3]. In [3], an estimation of $\Delta \varepsilon[k]$ is obtained by assuming the exponential tropospheric model in (6). Due to the small value of $n$, an EM refractivity index can be defined as $N = (n-1)10^6$. Then, the tropospheric model can be rewritten in function of the altitude as $N(h)=N_0e^{-bh}$, where $h$ is the altitude of the air platform carrying the radar in meters and $N_0$ is sea-level EM refractivity index. From geometrical considerations (the complete proof is given in [2] and [3]), the estimated elevation error is given by the following expression:

$$\Delta \varepsilon[k] = \frac{1}{2} \rho'^2 \left[ \frac{N_0b e^{-bh} \cdot 10^6}{1+N_0b e^{-bh} \cdot 10^6} \cos(\varepsilon'[k]) \right]$$

where $\rho'[k]$ and $\Delta \varepsilon[k]$ are the corrupted range and elevation measures. It’s important to note that such measures are considered free from the measurement noise, then, before using (9) we have to filter (e.g. with a Kalman filter) the radar measures.

4.2. Evaluation of the range error

As pointed out in [4], the atmospheric range error can be evaluated using the following path integral:

$$\Delta \rho = \int_{s_0}^{s_1} (n(s) - 1) ds,$$

where $n(s)$ is the refractive index and $ds$ is the infinitesimal element of the path between radar and target. Through a change of coordinates, the integral in (10) can be written as:

$$\Delta \rho = \int_{r_0}^{r_1} \frac{N(r) rdr}{\sqrt{r^2 - r_1^2 \cos^2 \varepsilon[k]}}.$$

where $r_1 = r_0 + h$, is the distance from the Earth’s centre to the radar ($r_0$ is the Earth’s radius and $h$ is the radar altitude, as pointed out before), $\varepsilon[k]$ is the target elevation angle at time $k$, $r_2[k]$ is the distance between the Earth’s centre and the target at time $k$, and $N(r)$ is the EM refractivity index as a function of $r$. The term $r_1$ is a priori known, since both Earth radius and radar altitude are constant. To evaluate the term $r_2$, we use the Carnot’s theorem:

$$r_2[k] = \sqrt{r_1^2 + \rho'[k]^2} + 2r_1 \rho'[k] \sin \varepsilon[k]$$

where $\rho'[k]$ is the corrupted range measure while $\varepsilon[k]$ is the true target elevation value. This means that, before evaluate the terms in (11) and (12), we must perform the elevation error correction using (9).
5. ATMOSPHERIC ERRORS CORRECTION

We describe here a modified version of the classical Kalman filter (KF). Our aim is to modify the KF equations in order to remove the atmospheric range and elevation errors from the estimated state vector, i.e. position and velocity vectors of the target. In the following, we propose an algorithm that allows us to estimate the correct state vector. This algorithm (Fig. 3) is composed of two parts: the aim of the first part is to obtain an estimate of the corrupted position vector free from the process and measurement noise (as the standard KF does), while the goal of the second part is to correct the atmospheric range and elevation errors. In the first part we apply the standard KF equations with unbiased measurements [5]. At the end of this first part, we have an estimate of the corrupted position vector, then we are able to evaluate the range and elevation errors from the corrupted position vector using (9) and (11). Finally, a new update equation of the state vector estimate is implemented in order to perform the atmospheric correction.

Figure 3. Algorithm for the atmospheric correction: the input is the converted measurement vector, while the output is the estimated state vector.

6. SIMULATION RESULTS

In this Section, some simulation results are illustrated. All the simulations have been performed using the radar parameters described in Sect. 2 for radar-target distance of about 90 km. The following figures represent the error mean value of the estimated z component of position (Fig. 4) and velocity (Fig. 5) vectors. The z component has been chosen as an example because it’s the component more heavily affected by the atmospheric errors. In fact, this component is affected by both range and elevation errors, whereas x and y components are affected only by range errors. We show here three different curves related to three different simulations: (1) the ideal case, i.e. the measurements have no atmospheric errors, (2) the case in which the measurements are affected by atmospheric errors and the correction of such errors is performed using the proposed KF-based algorithm, and (3) the case in which the measurements are corrupted but no correction is made. As we can see from Fig. 4, the atmospheric errors have a bias error of about 250 m in the worst case for the position, while for the z component of velocity the bias error is of about ±0.5 m/s in the worst case (Fig. 5). However, in both cases, the proposed algorithm is able to remove such bias error. Finally, it can be noted that the periodic-like behavior is due to the quasi-ellipsoidal aircraft race.

Figure 4. Mean value of the error for the z component of position.

Figure 5. Mean value of the error for the z component of velocity.

7. CONCLUSIONS

A KF-based algorithm has been proposed to correct the atmospheric range and elevation errors. The corrupted radar measures have been generated according to a mathematical model of the atmospheric propagation, then the atmospheric errors has been evaluated. A correction of the classical KF equations has been also implemented to remove such errors from the state vector estimate. Finally, the performances of the proposed algorithm have been investigated. Simulation results show the goodness of the proposed algorithm and its ability to reduce the effects of the atmospheric errors.

8. REFERENCES