Polarimetric Two-Scale Model

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ABSTRACT

We propose a polarimetric two-scale surface scattering model employed to retrieve the surface parameters of bare soils from polarimetric SAR data. The scattering surface is here considered as composed of slightly rough randomly tilted facets, for which the Small Perturbation Method holds. The facet random tilt causes a random variation of the local incidence angle, and a random rotation of the local incidence plane around the line of sight, which in turn causes a random rotation of the facet scattering matrix. Unlike other similar already existing approaches [1,2,7], our method considers both these effects. The proposed scattering model is then used to retrieve bare soil moisture and (large-scale) roughness from the co-polarized and cross-polarized ratios. The performances of the resulting retrieval algorithm is finally assessed by comparing obtained results to “in situ” measurements. To this aim, data from measurement campaigns available in literature are employed.

1. INTRODUCTION

Nowadays, several applications require the knowledge of many ground physical parameters (i.e., permittivity, ground roughness, soil moisture content, vegetation biomass-index, ecc…) relevant to wide natural areas and, of course, remote sensing technologies are the best candidates to provide these information in a comparatively short time. In fact, using multi-angle and/or multi-polarimetric Synthetic Aperture Radar (SAR) data allow us to estimate ground parameters, on condition that retrieval techniques are founded on reliable and realistic, but at the same time not too involved, models describing electromagnetic scattering phenomena. Accordingly, as almost all of already existing simple methods, like the Small Perturbation Method (SPM), do not take into account depolarization and cross-polarization effects, it is necessary to attain a new model which provides a good matching with measured data, just retaining an acceptable complexity.

In order to reach this aim, we devise a theoretical model to describe the scattering from a bare soil surface and, we use it to perform an effective inversion method to estimate both the dielectric constant $\varepsilon$ (or, in the same way, the soil moisture content $m_v$) and the ground roughness by building up numerical charts where co-polar and cross-polar ratios are plotted for different values of $\varepsilon$ ($m_v$) and of the roughness parameter $\sigma$, defined in Section 2. In our model we assume that the bare soil scattering surface is composed of slightly rough randomly-tilted facets, for which the SPM holds. Actually, such a random tilt causes a random drift of the local incidence angle and a random rotation of the local incidence plane (see Fig. 1 in the next page, where $h$ and $v$ respectively represent the orthogonal and the vertical components of the incident field, while $k$ identifies the incidence direction): at variance with other similar already existing approaches, we account for both these effects, starting from the stochastic description of the surface. We refer to our method as Polarimetric Two-Scale Model (PTSM).

2. THEORETICAL SETUP

We consider a bare soil surface as composed of large-scale variations on which a small-scale roughness is superimposed, so that we have a two-scale model of the surface. As regards the large-scale roughness, it is locally treated by replacing the surface with a slightly rough tilted facet, whose slope is the same of the smoothed surface at the center of the pertinent facet. Facets sizes are greater than the electromagnetic wavelength, but much smaller than sensor geometric resolution. We model both large- and small-scale roughness as stochastic processes, in fact we assume that the facet slopes along range and azimuth directions are independent identically distributed (i.i.d.) zero-mean $\sigma$-variance Gaussian random variables: this assumption only requires that the large-scale roughness is a Gaussian
statistically isotropic, stationary-increment process, so that it is compatible with both classical and fractal surface models.

\[
\begin{align*}
F_H &= \frac{\cos \vartheta_l - \sqrt{\epsilon - \sin^2 \vartheta_l}}{\cos \vartheta_l + \sqrt{\epsilon - \sin^2 \vartheta_l}} \\
F_V &= (\epsilon - 1) \frac{\sin^2 \vartheta_l - \epsilon (1 + \sin^2 \vartheta_l)}{(\epsilon \cos \vartheta_l + \sqrt{\epsilon - \sin^2 \vartheta_l})}
\end{align*}
\]

Once the square modulus of the scattered field is averaged over the facet (i.e., small-scale roughness), we can define the normalised radar cross sections (NRCS) for the single facet as:

\[
\sigma_{pq}^\beta = \frac{4 \pi^2 \langle |E_q^\beta| \rangle^2}{A |E_p|} = \frac{4 k^4 \cos^4 \vartheta_l}{\pi} \chi_{pq}^\beta \tilde{\delta}(0, 2k \sin \vartheta_l),
\]

where \( W(\kappa) \) is the power spectral density of \( \delta(x,y) \), \( \Lambda \) is the facet’s area and the symbol \( \langle f \rangle_{\xi} \) stands for “the mean of \( f \) with respect to the random variable \( \xi \)”. If the large-scale roughness height variations are larger than the wavelength and the facet size is larger than small-scale roughness correlation length, then the returns from different facets are uncorrelated, and NRCS of the whole surface can be obtained by averaging those of a single facet over \( \beta \) and \( \vartheta_l \) or, equivalently, over the azimuth and range slopes \( a \) and \( b \), related to \( \beta \) and \( \vartheta_l \) by the following relations [3]:

\[
\cos \vartheta_l = \frac{\cos \vartheta + b \sin \vartheta}{\sqrt{1 + a^2 + b^2}}, \quad \tan \beta = \frac{a}{b \cos \beta + \sin \vartheta}.
\]

Since such statistical averages cannot be evaluated analytically in a closed form, then, assuming small values for facet slopes, the Taylor expansion of NRCS around \( a=0, b=0 \) can be used. Accordingly, recalling that \( a \) and \( b \) are i.i.d. zero-mean \( \sigma^2 \)-variance Gaussian random variables and by using the formula to obtain moments of any order for a zero-mean Gaussian variable \( Z \sim N(0, \sigma^2) \)

\[
\langle Z^n \rangle = \begin{cases} 
0, & \text{if } n \text{ is odd}
\end{cases}
\]

\[
\begin{cases} 
1 \cdot 3 \cdot 5 \cdot \ldots \cdot (n-1) \sigma^n, & \text{if } n \text{ is even}
\end{cases}
\]

it is possible to compute averages of NRCS for the whole scattering surface:

\[
\langle \sigma_{pq}^\beta(a,b) \rangle_{a,b} = \sum_{n=0}^{\infty} \frac{1}{n!} \left[ \sum_{k=0}^{n} \binom{n}{k} \frac{\partial^n \sigma_{pq}^\beta}{\partial a^k \partial b^{n-k}} \right] \langle a^k \rangle \langle b^{n-k} \rangle.
\]
Considering Taylor expansion terms up to the second order, this leads to express normalised radar cross sections simply as:

\[
\sigma_{pq}(a, b)_{a,b} = C_{pq}^{0} + \left( C_{pq}^{2} + C_{pq}^{0,2} \right) \sigma^2,
\]

in which \( C_{pq}^{k,n-k} \) are the series expansion coefficients, defined as

\[
C_{pq}^{k,n-k} = \frac{1}{n!} \left( \frac{\partial^n \sigma_{pq}^0}{\partial a^n \partial b^{n-k}} \right)_{a=b=0}.
\]

3. RESULTS AND METHOD VALIDATION

Once the NRCS are computed, they can be used to built up numerical charts based on the co-polar and cross-polar ratios (i.e., the ratios between \( \sigma_{vv}^0 \) and \( \sigma_{hh}^0 \) and between \( \sigma_{hv}^0 \) and \( \sigma_{vh}^0 \), respectively), parameterized by the dielectric constant \( \varepsilon \) (or the soil moisture content \( m_v \)) and the large-scale roughness \( \sigma \). These charts, an example of which is shown in Figure 2, can be used to get the soil moisture content and the large-scale roughness from a pair of co-pol, cross-pol measured data, obtained for instance using polarimetric SAR images.

Figure 2: PTSM based Co-pol Cross-pol chart for \( \theta=45^\circ \)

Of course, this approach can be performed in an unsupervised way, simply making use of special purpose look-up software, able to create soil moisture maps of sensed scenes just comparing processed input SAR data with correspondence tables built up like above mentioned charts.

In order to validate our retrieval method, a wide variety of scattering data at different frequencies, incidence angles, surface roughness and soil moisture contents, in conjunction with the corresponding ground truth, has been used: in practically all cases, results obtained by using the PTSM have turned out to be in better agreement with measurements than those obtained by using already existing similar models. For instance we consider L-band AIRSAR data acquired on several different days during a measurement campaign in the Little Washita basin, in June 1992 [5]. We select the bare soil field labeled as AG002 in [5], for which the volumetric soil moisture content was monitored “in situ”, and we perform on it the soil moisture retrieval using the above mentioned procedure. The retrieved values of the relative dielectric constant are converted into volumetric moisture values \( m_v \) by using the mixing model of [6], with percentages of sand and clay equal to 45.5 and 13.4, respectively [5]. The retrieved results for PTSM are reported in Table I, together with in situ measured values and with results obtained by using the model of [1] (X-Bragg). It is clear that better results are obtained via the PTSM rather than via the X-Bragg.

<table>
<thead>
<tr>
<th>Day</th>
<th>( m_v ) X-SPM</th>
<th>( m_v ) PTSM</th>
<th>( m_v ) In situ</th>
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<tr>
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<td>0.140</td>
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<td>0.102</td>
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8. REFERENCES