STUDY OF GROUND-BASED SAR IMAGING ALGORITHMS

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ABSTRACT

In the last decade, Ground-Based Synthetic Aperture Radar (GB-SAR) systems have gained an increasing interest. Many different applications take advantages of the flexibility and capability of such systems to measure deformations with submillimeter precision, map terrains and monitor landslides and other geological phenomena. In literature different focusing algorithms have been proposed. The aim of the paper is to provide a comparative analysis of the aforementioned focusing algorithms for GB-SAR systems.

1. INTRODUCTION

GB-SAR is a relatively new system employed in short ranges and frequent monitoring applications [1, 2]. It provide some advantages with respect to space and air-borne SAR sensors such as higher image resolution and shorter revisiting time. A single image is acquired in few minutes, while the maximum range of acquisition is kept within 4 km. Furthermore, the capability of GB-SAR interferometers to measure deformations with a sub-millimeter accuracy together with their flexibility makes them a useful tool for monitoring landslides and other geological phenomena in emergency cases. The acquisition process is the passage of the reflectivity map through the system impulse response [3]. The synthesis of a SAR image is the reverse process: a correlation between the data and the space-variant, matched in phase impulse response function but it cannot be directly implemented because of its computational inefficiency. In literature other focusing algorithms have been proposed for GB-SAR image formation, such as Frequency-Domain Back-Propagation, Wavenumber-Domain and Time-Domain Algorithms [3]. Moreover, the compressive sensing theory can be also employed [4, 5] for GB-SAR focusing: it exploits the undersampling of the data letting the image generation process become faster.

The focusing algorithms have been tested on a simulated scene in order to compare them in terms of image quality and computational time.

2. GROUND BASED SAR SYSTEM AND ACQUISITION GEOMETRY

A GB-SAR (see Fig.1) consists in a stepped frequency continuous wave (SF-CW) radar moved along a rail of finite length, that changes its position of a constant step, while a burst of pulses characterized by different progressive frequencies are transmitted and the correspondent backscattering echoes received. Since the synthetic aperture is realized by the antenna movement along the finite length rail, it results of a limited amount; at the same time, the extension of the area of interest illuminated by the antenna, is limited, too (up to few kilometers). The range resolution \(dp\) is function of the system bandwidth \(B\) (\(dp = c/2B\)), being \(c\) the speed of light, while the azimuth resolution worsens with increasing range distance and also depends on the angular position and rail length.

In each position of the rail \(x_n\), the transmitted train of sinusoids is:

\[
s_T(x_n, f_m, t) = \exp(j2\pi f_{m}t) \quad (1)
\]

If \(P\), a single point scatterer positioned in \((\rho_P, \varphi_P)\) is in the scene, the returned signal after coherent demodulation is:

\[
s_R(x_n, f_m, t) = \exp \left( -j4\pi f_m \frac{R}{c} \right) \cdot \exp \left( -j2\pi f_m \left( t - \frac{2R}{c} \right) \right) \quad (2)
\]

Range compression is achieved by coherently summing the signal contributions relative to the different stepped frequencies, that is equivalent to a discrete inverse Fourier transform. The system impulse response is then [3]:

\[
h(x, t; \rho, \varphi) = p \left( t - \frac{2R}{c} \right) \exp \left( -j4\pi f_c \frac{R}{c} \right) \quad (3)
\]

For a complex scene characterized by a reflectivity map \(a(\rho, \varphi)\), the acquisition process consists in the passage of the reflectivity map through the system impulse response:

\[
d(x, t) = \int \int a(\rho, \varphi) h(x, t; \rho, \varphi) \, d\rho d\varphi \quad (4)
\]
3. IMAGING ALGORITHMS

The focusing is mathematically achieved by inverting Eq. (4) but can not be practically employed because of an unnecessary oversampling of the focused data.

\[
a(\rho, \varphi) = \int \int d(x,t)h^*(x,t; \rho, \varphi) \, dp \, d\varphi
\]  

(5)

An imaging technique is the Frequency-Domain Back-Propagation (FDBA) that performs a coherent sum of the different frequency contributions for each radar position, corrected for the phase delay. Although it is an exact focusing method its implementation is very heavy.

\[
a(\rho, \varphi) = \sum_m \sum_n d(x_m, f_m) \exp\left(j4\pi f_m \frac{R}{c}\right)
\]  

(6)

Alternatively the space-variant correlation in Eq. (4) can be performed in the 2D transformed domain, by multiplying the system transfer function and the Fourier transform of the range compressed data.

\[
a(\rho, \varphi) = \mathcal{F}^{-1}\left(D(k_x, \omega; \rho, \varphi) \cdot H^*(k_x, \omega; \rho, \varphi)\right)
\]  

(7)

The use of Stolt interpolation is necessary to separate the 2D Fourier kernel in the \((k_x, \omega)\) domain, to incorporate the range-variance of the impulse response. This technique does not uses approximations (apart for the stationary principle to compute the Fourier transform of the range compressed data) but it has two major drawbacks: the 2D mapping interpolation and the high computational cost.

Approximated solutions have been proposed in literature ensuring computational efficiency. A time domain technique ignores the range migration and assumes the far-field condition leading to a simplified impulse response and a space-invariant focusing. The focused data are

\[
a(\rho, \varphi) \approx d(x, \rho) \exp\left(j\omega_c \frac{2}{c} \left(\rho + \frac{x^2}{2\rho} - xsin\varphi\right)\right)
\]  

(8)

\[
\text{Table I: GB-SAR acquisition and object space geometry}
\]

<table>
<thead>
<tr>
<th>Method</th>
<th># multpls</th>
<th>Numeric value / output sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>FDBA</td>
<td>(O\left(M^3N^3\right))</td>
<td>(\approx 2.14 \times 10^6)</td>
</tr>
<tr>
<td>WNDA</td>
<td>(O(M^2N^2) + KMN'))</td>
<td>(\approx 3 \times 10^7)</td>
</tr>
<tr>
<td>TDA (=FPFA, (p = 0))</td>
<td>(O(MN log_2(MN) + KMN'))</td>
<td>(\approx 100)</td>
</tr>
<tr>
<td>FPFA, (p \neq 0)</td>
<td>(O((p + 1)(MN log_2 MN + \frac{3pMN}{K} + KMN')))</td>
<td>(\approx 3 \times 10^8, p = 2)</td>
</tr>
</tbody>
</table>

that can be interpreted as a multiplication and a Fourier transform:

\[
a(\rho, \varphi) \cong \exp\left(j\omega_c \frac{2}{c} \left(\rho + \frac{x^2}{2\rho} - xsin\varphi\right)\right) \mathcal{F}\left(d(x, \rho) \exp\left(j\omega_c \frac{x^2}{c}\right)\right)
\]  

(9)

The computational cost associated to the algorithms is summarized in Table I, being \((M, N)\) the dimensions of the raw data matrix and \((M', N')\) that of the focused image (here \(M' = M\) is considered).

4. COMPRESSION SENSING FOR GBASR IMAGING

The compressive sensing (CS) technique can also be used to perform GBASR imaging by exploiting a reduced number of samples both of azimuth positions and frequencies. It enables the reconstruction of sparse signals from a small set of nonadaptive linear measurements.

Given a signal vector of interest, \(x\), with length \(N\), it is \(K\)-sparse in an orthogonal basis \(\Psi\) if the projection vector \(z = \Psi x\) has \(K\) elements different from zero. Moreover, if a vector of \(M\) measurements \(y\) is available by projecting the signal vector onto \(M\) random basis functions \(\Phi\), i.e., \(y = \Phi x\), then it holds:

\[
y = \Phi x = \Phi \Psi^H z = \Theta z
\]  

(10)

and \(z\) can be reconstructed by solving a L1 optimization problem that returns the number of nonzero elements in the vector: \(\ell_1\)-norm \(\|z\|_1\) s.t \(y = \Theta z\).

It is however necessary that two conditions hold. The sensing matrix \(\Phi\) and the orthogonal basis \(\Psi\) must be mutually incoherent: this ensure that the signal information is insensitive to the downsampling. Additionally, the mapping matrix \(\Theta\) must satisfy the restricted isometry property, which means that all its submatrices composed by \(K\) non-zero columns should be nearly orthogonal.

In order to exploit the CS technique for GBASR imaging first a sparse scene is simulated as a discrete reflectivity map \(a\) with only few targets and dimension \((S \times T)\). Then the received signal in Eq. (2) is discretized and express in a matrix notation as follows:

\[
s(m, n) = \sum_{i=1}^{S \times T} a(i) \cdot \exp\left(-j4\pi f_m \frac{R(m, n, i)}{c}\right)
\]  

\[
s = \Theta z
\]  

(11)
with $\Theta$ the $MN \times ST$ mapping matrix, $M$ the total number of frequencies and $N$ the azimuth steps along the rail. If elements from $s$ and, accordingly, $\Theta$ are randomly selected and deleted the Nyquist sampling requirements may not be satisfied but the reconstruction is carried out by applying the CS scheme to the reduced form if the sparsity condition hold. The employed reconstruction algorithm is the regularized orthogonal matching pursuit [6].

5. FOCUSING METHODS COMPARISON

The above considered algorithms have been tested on simulated scenes including one or more scatterers. The wavenumber domain algorithm (WNDA) performances are always superior to those of Time Domain one (TDA) both for along and across tracks, in terms of Peak-to-Sidelobe Ratio (PSLR), Integrated-to-Sidelobe Ratio (ISLR) and -3dB main lobe and also as it is clear by observing Fig.2. In Table II the performance parameters are summarized by averaging their values obtained for simulated targets at different range distances. Moreover, by adding clutter in the scene TDA provides worse results than those without clutter, while they remain almost constant by using the WNDA.

Concerning the reconstructed scene by using the CS, it is exactly equal to the simulated one if there are only one or two targets. This applies for any ratio of selected samples over the total ones. On the contrary, if there are more than two targets in the scene, the recovered signal differs from the original one. Fig. 3 shows the mean square error (MSE) obtained by varying the undersampling ratio, which is the portion of the selected samples, and the sparsity level of the scene corresponding to the number of targets in it. Specifically, the errors that occur are target mispositioning, target amplitude variation and aliasing.

6. CONCLUSIONS

In this paper theoretical aspects of the focusing have been reported in order to compare them by using simulated scenes. The target quality has been evaluated by means of the resolution, peak and integrated sidelobe ratios, both in presence or in absence of underlying clutter. The Time Domain Algorithm can be achieved in a very simple way (windowing and a double Fourier transform) and is well suited for the far field case with also a lower computational time. On the contrary, the Wavenumber Domain Algorithm is the appropriate choice to process images acquired in the near–field where the TDA is not able to provide optimal quality, even if at the cost of an higher computational time.

On the other side, the preliminary use of the compressive sensing in the framework of Ground-Based SAR data imaging has shown that this technique could be fully exploited in the GBSAR data processing framework. In order to understand its real potentialities for real applications, further studies need to be done to reduce the errors and to extended the processing to real scenes.

7. REFERENCES


